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Hadron Electric Dipole Moments from CP-Odd Operators of Dimension Five Via QCD Sum Rules: The Vector Meson

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Abstract

We present a complete analysis of the electric dipole moment (EDM) of the ρ-meson induced by CP violating operators of dimension 4 and 5 within the QCD sum rules approach. The set of CP-odd operators includes the theta term and the electric and chromoelectric dipole moments of the three light quarks. We find that the ρ-meson EDM induced purely by the EDMs of quarks is smaller, but still in reasonable agreement, with the predictions of a naive constituent quark model. However, the chromoelectric dipole moments, including that of the strange quark, give comparable and sometimes larger contributions. We also consider the effect on the hadronic EDM of the existence of Peccei-Quinn symmetry. When this symmetry is active, chromoelectric dipole moments induce a linear term in the axion potential which leads to a numerically important vacuum contribution to the hadronic EDM.

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I. INTRODUCTION

Tests of time-reversal symmetry at low energies are an important source of information about the CP properties of the physics at and above the electroweak scale, complementary to that coming from K and B meson physics. Impressive experimental progress achieved during the last decade has brought the limits on the electric dipole moments (EDMs) of neutrons, heavy atoms, and molecules [1] down to a remarkably low level. The Kobayashi-Maskawa model, so successful in explaining the observed CP violation in K mesons, predicts EDMs to be several orders of magnitude smaller than the current experimental sensitivity. This presents a unique opportunity for limiting extra sources of CP-violation, and the constraints resulting from EDM data are generally very strong [2].

Generically, EDMs can be used to probe the physics at a high energy scale by limiting the coefficients of operators $\mathcal{O}_i$ with dimension $k \geq 4$ in the effective low energy Lagrangian. The effective Lagrangian for these operators has the form,

$$\mathcal{L}_{eff} \sim \sum_i c_i M^{4-k} \mathcal{O}_i^{(k)}, \tag{1}$$

where M is the mass scale at which these effective operators are induced and c_i their coefficients which, in general, have logarithmic scale dependence. These operators are odd under CP transformations and their coefficients c_i are proportional to the fundamental CP-violating phases of the underlying theory. Consequently, the calculation of a hadronic EDM can naturally be separated into two main parts. Firstly, there is the calculation of the coefficients c_i for a specific model of CP-violation which involves integrating out distances shorter than M^{-1}. The second part, which is by far the more complicated, is the problem of switching from the perturbative quark-gluon description to the level of hadrons which requires nonperturbative input.

In this letter, we present a systematic analysis of the EDM of the simplest hadron – the ρ-meson (more specifically ρ^+) – induced by operators with dimension 4 and 5 within QCD sum rules which up to now remains the most reliable analytical method for calculating the properties of hadrons [3]. We choose the following parametrization of dimension four and five CP violating sources:

$$\delta\mathcal{L} = \bar{\theta}\frac{\alpha_s}{8\pi} G\tilde{G} - \frac{i}{2}\sum_{q=u,d,s} d_q \overline{q} F\sigma\gamma_5 q - \frac{i}{2}\sum_{q=u,d,s} \tilde{d}_q \overline{q} g_s G\sigma\gamma_5 q, \tag{2}$$

where $F_{\mu\nu}$ and $G_{\mu\nu}$ are the electromagnetic and gluonic field strength tensors. The first term here is the effective theta term, which primarily is due to the fundamental QCD vacuum angle θ_{QCD}. This represents a challenge for particle physics model building as naturally one would expect $\bar{\theta} \sim O(1)$ which is ruled out by all available data on EDMs. As a consequence, one is usually led to introduce the Peccei-Quinn (PQ) mechanism via which this primary source of $\bar{\theta}$ is removed, $\theta_{QCD} = 0$. Nonetheless, even in the presence of PQ symmetry, as we shall discuss, the effective θ-term is non-vanishing. The two sums in (2) represent the EDMs and chromoelectric dipole moments (CEDMs) of the three light quark flavors. Other quarks are considered to be heavy and can thus be integrated out producing operators of dimension 6 and higher.

To motivate this calculation, we note that in principle experiments [1] impose strong constraints on some combination of the coefficients $\bar{\theta}$, d_i, and $\tilde{d}_i$ and thus on the fundamental CP-phases of the theory. In order to extract these constraints, however, we have to embark on a non-perturbative calculation of the electric dipole moment of the neutron in particular (or nucleon-nucleon interaction, as in the case of the EDM of ^{199}Hg) induced by the effective Lagrangian (2). The connection between different EDM observables and the coefficients $\bar{\theta}$, d_i, $\tilde{d}_i$ is especially important in the framework of supersymmetric theories where eq. (2) and an additional three-gluon CP-odd operator [5] represent a complete set of the relevant operators with the coefficients explicitly calculable as functions of the soft-breaking parameters. (For a recent discussion in the context of the MSSM, see e.g. [6]).

Generically, the EDM of a hadron can be written as a linear combination of the coefficients in eq. (2)

$$d_h = d_h(\bar{\theta}) + d_h^{\mathrm{EDM}}(d_u, d_d, d_s) + d_h^{\mathrm{CEDM}}(\tilde{d}_u, \tilde{d}_d, \tilde{d}_s). \tag{3}$$

In the case of the neutron EDM, $d_n(\bar{\theta})$ has been calculated using various different techniques: (1) making use of the dominance of a pion loop-induced logarithm in the chiral limit [7]; (2) in the Skyrme model [8]; and finally using QCD sum rules [9,10]; and all have produced similar results. For the calculation of $d_n^{\mathrm{EDM}}(d_u, d_d, d_s)$, the quark tensor charges over the nucleon are required, and various techniques [11,12] have produced results consistent with the predictions of a naive SU(6) quark model. We note that the contribution of the strange quark EDM is found to be consistent with zero in all methods.

Unfortunately, the quantitative evaluation of the CEDM contribution to the neutron EDM is considerably more complicated and although a number of serious attempts have been made to estimate $d_n^{\mathrm{CEDM}}(\tilde{d}_u, \tilde{d}_d)$, these results often differ by more than one order of magnitude. In particular, direct QCD sum rules calculations [13,14] give $d_n \sim 20$ times smaller than the estimates based on the chiral loop approach [15,16]. Thus, we believe that an independent calculation of $d_n^{\mathrm{CEDM}}(\tilde{d}_u, \tilde{d}_d, \tilde{d}_s)$ within the QCD sum rule approach is absolutely necessary in order to clarify the magnitude and sign of the contribution of quark CEDMs to the EDM of the neutron.

With this motivation in mind, the discussion of the ρ-meson presented here[1] will serve a dual purpose. Firstly, the detailed analysis of the contribution of dimension four and five CP-odd operators to a particular hadronic EDM allows us insight into the relative sizes of the contributions, and in particular the dominant mechanism inducing the EDM, which may well prove quite universal. Secondly, the ρ-meson is the simplest light quark system where QCD sum rules are known to work well. Thus it is a convenient arena in which to develop techniques that should later prove useful in an analogous, but much more involved, study of nucleon sum rules and the neutron EDM. In this regard, we recently calculated $d_\rho(\theta)$ by generalizing the operator product expansion (OPE) to the case of an external θ background [18], which was subsequently applied to the case of $d_n(\theta)$ [9]. In this letter this method is developed further to include all CP-odd dimension 5 operators. As a result we obtain d_ρ as an explicit function of $\bar{\theta}$, d_i, and $\tilde{d}_i$. We observe the numerical importance of

[1]Note that the ρ-EDM was also studied within the context of a quark model [17].

all CEDM contributions and a strong dependence of the result on the existence/absence of PQ symmetry. With the use of the techniques developed here and in our earlier work, the question of $d_n^{\rm CEDM}(\tilde{d}_u, \tilde{d}_d, \tilde{d}_s)$ will be readdressed in the near future [19].

II. OPE ANALYSIS

Within the sum rule approach to the ρ^+ EDM, we need to consider the correlator of currents with ρ^+ quantum numbers, in a background with a CP violating source and an electromagnetic field $F_{\mu\nu}$,

$$\Pi_{\mu\nu}(-p^2) = i\int d^4x e^{ip\cdot x}\langle 0|T\{j_\mu^+(x)j_\nu^-(0)\}|0\rangle_{\not{C}\not{P},F}. \tag{4}$$

In the presence of these sources, and since we will work outside the chiral limit with unequal quark masses, it is necessary to take into account mixing between the vector current associated with ρ^+ and the axial-vector current [18]. Thus we parametrize the full current in form

$$j_\mu^+ = V_\mu + icA_\mu, \tag{5}$$

where c is a (real) mixing parameter to be determined, and

$$V_\mu = \overline{u}\gamma_\mu d \qquad A_\mu = \overline{u}\gamma_\mu\gamma_5 d. \tag{6}$$

The parameter c is linear in the CP violating source, and thus to first order we have,

$$\Pi_{\mu\nu} = \langle V_\mu V_\nu^\dagger\rangle_{\not{C}\not{P}} + ic(\langle V_\mu A_\nu^\dagger\rangle - \langle A_\mu V_\nu^\dagger\rangle) + \cdots \tag{7}$$

In evaluating these correlators, we require the propagator in the presence of CP violating sources, $S_{ab}^q \equiv \langle q_a(x)\overline{q}_b(0)\rangle_{\not{C}\not{P},F}$, which has the form

$$\int d^4x e^{ip\cdot x}S_{ab}^q(x) = i\frac{(\not{p}+m\mathbf{1})_{ab}}{p^2} - \frac{d_q}{2}\frac{(\not{p}F\sigma\gamma_5\not{p})_{ab}}{p^4} - \frac{\tilde{d}_q}{2}\frac{(\not{p}gG\sigma\gamma_5\not{p})_{ab}}{p^4}. \tag{8}$$

Using eq. (8), we can express the relevant current correlators as

$$\int d^4x e^{ip\cdot x}\langle 0|V_\mu(x)V_\nu^\dagger(0)|0\rangle = -i\int d^4x e^{ip\cdot x}{\rm Tr}(\gamma_\mu S^d(x)\gamma_\nu S^u(-x)), \tag{9}$$

with similar expressions for $\langle VA^\dagger\rangle$ and $\langle AV^\dagger\rangle$. The appropriate tensor structure to isolate is $\tilde{F}_{\mu\nu}(=\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}/2)$, and we evaluate the correlators as usual by projecting to the vacuum in order to extract this structure (see e.g [18] for more details in the case of ρ^+). In doing so we will assume a constant background electromagnetic field, and use a fixed point gauge [20] for the gluon field. Condensates involving quark and gluon fields are then parametrized in terms of certain "condensate susceptibilities" following Ioffe and Smilga [21],

$$\begin{aligned} g_s\langle 0|\overline{q}G\sigma q|0\rangle &= -m_0^2\langle 0|\overline{q}q|0\rangle \\ \langle 0|\overline{q}\sigma_{\mu\nu}q|0\rangle_F &= e_q\chi F_{\mu\nu}\langle 0|\overline{q}q|0\rangle \\ g_s\langle 0|\overline{q}(G_{\mu\nu}^a t^a)q|0\rangle_F &= e_q\kappa F_{\mu\nu}\langle 0|\overline{q}q|0\rangle \\ 2g_s\langle 0|\overline{q}\gamma_5(\tilde{G}_{\mu\nu}^a t^a)q|0\rangle_F &= ie_q\xi F_{\mu\nu}\langle 0|\overline{q}q|0\rangle. \end{aligned} \tag{10}$$

FIGURES

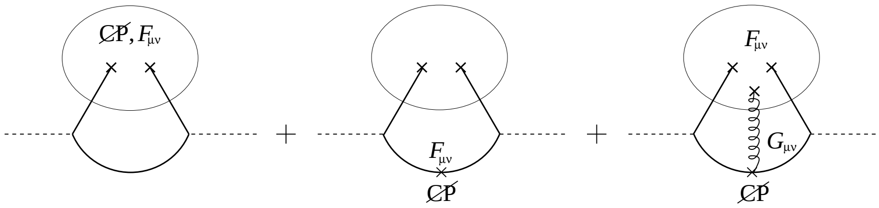

FIG. 1. Contributions to the correlator at leading order in $F_{\mu\nu}$.

In general, the OPE will contain perturbative pieces in addition to terms explicitly proportional to the quark-gluon condensates. However, it turns out that the simple perturbative contributions are heavily suppressed. This is due to the chirality-flipping structure of the EDM and CEDM operators. The result then vanishes unless there is an additional chirality flip in the loop which can only come from a quark mass in the propagator. Indeed, for the θ-induced contribution, the perturbative piece is identically zero [18].

Thus, we are left with only the non-perturbative contributions, for which the leading order diagrams are given in Fig. 1. Naively, one could drop all the terms in the quark propagator which are proportional to a small mass, m_u or m_d. This is not valid, however, as it turns out that at different stages of the OPE calculation, quark masses from the propagator, $m_{u(d)}$, enter in front of vacuum correlators of the form $C \sim \int d^4x \langle 0|T(\overline{u}\gamma_5 u, \bar{u}(G\sigma)\gamma_5 u(x)|0\rangle$. These correlators can be saturated by pion exchange, producing a non-vanishing contribution in the chiral limit, $m_q C \sim O(m_q/m_\pi^2) \to$ const. Equivalently, one can perform a chiral rotation, requiring that pions cannot be produced from the vacuum, $\langle 0|\mathcal{L}_{eff}|\pi_0\rangle = 0$. This leads to the appearance of additional γ_5-mass terms in the Lagrangian which will certainly contribute to d_ρ. We prefer, however, not to perform the chiral rotation and instead to account for the vacuum contributions explicitly.

Using this approach, we find for $\langle VV^\dagger\rangle$,

$$\langle V_\mu(p)V_\nu^\dagger(0)\rangle = \frac{i\tilde{F}_{\mu\nu}}{p^2}\Big(i\chi\left(m_d e_u\langle\overline{u}\gamma_5 u\rangle_{\not{C}\not{P}} - m_u e_d\langle\overline{d}\gamma_5 d\rangle_{\not{C}\not{P}}\right) + (d_d - d_u)\langle\overline{q}q\rangle - \left(\kappa - \frac{\xi}{2}\right)(e_d\tilde{d}_u - e_u\tilde{d}_d)\langle\overline{q}q\rangle\Big), \tag{11}$$

where the first term arises from Fig. 1(a), the second from Fig. 1(b), and the third from Fig. 1(c). A useful simplification in these calculations follows by noting that, although naively contributing to the structure $\tilde{F}_{\mu\nu}$ via the equations of motion, contributions arising from a Taylor expansion of the quark wave function, actually vanish! Thus we find the rather compact expression exhibited in Eq. (11).

The necessity for additional contributions, due to mixing with the axial vector current, now becomes clear. Indeed, if eq. (11) were the entire answer, the first line calculated in the external θ background would produce an incorrect quark mass dependence, differing from $m_u m_d/(m_u + m_d)$. Thus its clear that we need to address mixing with the axial current [18]. The mixing parameter c may be obtained to leading order by considering the correlators $\langle VV^\dagger - AA^\dagger\rangle$, and $\langle VA^\dagger - AV^\dagger\rangle$ with the external field turned off. At this order we may diagonalize on the tensor structure $g_{\mu\nu}p^2$, and we obtain

$$c = \frac{1}{2}\left[\frac{i}{6}(\tilde{d}_d - \tilde{d}_u)\frac{m_0^2}{m_u+m_d} - i\frac{m_d\langle\overline{u}\gamma_5 u\rangle_{CP\!\!\!/} - m_u\langle\overline{d}\gamma_5 d\rangle_{CP\!\!\!/}}{(m_u+m_d)\langle\overline{q}q\rangle}\right]. \tag{12}$$

This value of c ensures the absence of mixing to leading order between the "eigen-currents" $V_\mu + icA_\mu$ and $iA_\mu + cV_\mu$. The calculation of the mixed correlators is straightforward, and the result is given by

$$\langle VA^\dagger\rangle = -\langle AV^\dagger\rangle = -\frac{\tilde{F}_{\mu\nu}}{p^2}\chi(m_u e_d + m_d e_u). \tag{13}$$

Combining the relevant pieces according to eq. (7) we arrive at the following result:

$$\Pi_{\mu\nu} = \frac{i\tilde{F}_{\mu\nu}}{p^2}\langle\overline{q}q\rangle\left[d_d - d_u + \chi(e_d - e_u)\frac{m_u m_d}{m_u+m_d}\frac{\langle\bar{d}\gamma_5 d + \bar{u}\gamma_5 u\rangle_{CP\!\!\!/}}{\langle\overline{q}q\rangle} + \right.$$
$$\left.\chi m_0^2(\tilde{d}_d - \tilde{d}_u)\frac{e_u m_d + e_d m_u}{m_u+m_d} + \left(\kappa - \frac{\xi}{2}\right)(\tilde{d}_d e_u - \tilde{d}_u e_d)\right] \tag{14}$$

The next problem to address is the calculation of the vacuum matrix elements in eq. (14). These terms, arising from Fig. 1(a), require the evaluation of correlators of the form $\int d^4y\langle\overline{q}\gamma_5 q(x), i\delta\mathcal{L}(y)\rangle$, where $\delta\mathcal{L}$ given in Eq. (2) involves in particular the θ-term and the colour EDM sources which may be extracted from the vacuum at leading order in the background electromagnetic field. The case of θ was discussed at length in our previous paper [18]. Here we shall concentrate on the CEDMs. We will evaluate these correlators by inserting a complete set of intermediate states $\{\epsilon_i\}$,

$$\langle\overline{q}\gamma_5 q(x), i\delta\mathcal{L}(y)\rangle = \langle\overline{q}\gamma_5 q(x)\left(\sum_i |\epsilon_i\rangle\frac{i}{-m_i^2}\langle\epsilon_i|\right) i\delta\mathcal{L}(y)\rangle. \tag{15}$$

For the two-flavour case, to a good approximation we need only consider π_0, while for three flavours we also include η. The neglected states then have masses ≥ 1GeV and will remain massive in the chiral limit. They may contribute corrections of no more than 25%, which we shall factor into our precision estimate. The remaining condensates may be reduced to commutators in a manner analogous to the soft pion theorem in chiral perturbation theory. For a generic operator $\mathcal{O}$ we have

$$\langle 0|\mathcal{O}|\pi_0\rangle = \frac{i}{2f_\pi}\langle 0|[\mathcal{O}, u^\dagger\gamma_5 u - d^\dagger\gamma_5 d]|0\rangle \tag{16}$$
$$\langle 0|\mathcal{O}|\eta\rangle = \frac{i}{2\sqrt{3}f_\pi}\langle 0|[\mathcal{O}, u^\dagger\gamma_5 u + d^\dagger\gamma_5 d - 2s^\dagger\gamma_5 s]|0\rangle. \tag{17}$$

Concentrating on the colour EDM sources in (2) we then obtain

$$\langle\overline{u}\gamma_5 u\rangle_{\pi_0} = -\langle\overline{d}\gamma_5 d\rangle_{\pi_0} = \frac{i}{2}(\tilde{d}_u - \tilde{d}_d)\langle\overline{q}q\rangle\frac{m_0^2}{m_u+m_d} \tag{18}$$
$$\langle\overline{u}\gamma_5 u\rangle_\eta = \langle\overline{d}\gamma_5 d\rangle_\eta = -\frac{i}{4}\tilde{d}_s\langle\overline{q}q\rangle\frac{m_0^2}{m_s} + \cdots, \tag{19}$$

where in the last expression we have neglected terms of $O(\tilde{d}_{(u,d)}/m_s)$, assuming an approximate proportionality of the CEDMs to the quark masses, i.e. $\tilde{d}_d/\tilde{d}_s \sim m_d/m_s \ll 1$.

Putting the pieces together, and decomposing into singlet and triplet combinations, we finally obtain the following result for the correlator in (7),

$$\Pi_{\mu\nu} = \frac{i\tilde{F}_{\mu\nu}}{p^2}\langle\overline{q}q\rangle \left[-\frac{\chi}{2}e_- m_* \bar{\theta} + d_- \right. \\ \left. + \frac{\chi m_0^2}{12}\tilde{d}_-(e_+ - e_-\tilde{m}) + \frac{\chi m_0^2}{4}\tilde{d}_s e_- \frac{m_*}{m_s} + \frac{1}{2}\left(\kappa - \frac{\xi}{2}\right)(e_-\tilde{d}_+ - e_+\tilde{d}_-) \right], \tag{20}$$

where we have introduced the notation: $d_\pm = d_u \pm d_d$, $\tilde{d}_\pm = \tilde{d}_u \pm \tilde{d}_d$, $e_\pm = e_u \pm e_d$, $m_* = m_u m_d/(m_u + m_d)$, and $\tilde{m} = (m_u - m_d)/(m_u + m_d)$.

III. THE EDM AND PECCEI-QUINN SYMMETRY

The phenomenological side of the sum rule for the correlator $\Pi_{\mu\nu}$ may be obtained from the form-factor Lagrangian, $\mathcal{L} = -2id_\rho m_\rho \rho_\mu^+ \tilde{F}_{\mu\nu}\rho_\nu^+ + \cdots$. We find,

$$\Pi_{\mu\nu}^{\text{Phen}} = -2id_\rho m_\rho \tilde{F}_{\mu\nu}\frac{\lambda^2}{(p^2 - m_\rho^2)^2} + \cdots, \tag{21}$$

where λ is the coupling to ρ^+, and we have kept only the double pole term. We now follow standard practice, equating this result with (20) and performing a Borel transform to suppress the contribution of continuum states. Rather than presenting a detailed analysis of the sum rule, including a parameterization of single pole and continuum states, we shall consider only the double pole term here, as previous work [18,9] indicates that the corrections which arise from a more careful analysis are overwhelmed by errors from other sources. At this order, the coupling may be obtained in terms of the Borel mass M as $\lambda^2 = m_\rho^2 M^2/4\pi^2$ from the CP even sum rule (see e.g. [22]). It turns out that within these approximations d_ρ does not depend on M^2. We then obtain the following result for the EDM induced by $\overline{\theta}$, (d_u, d_d) and the colour sources $(\tilde{d}_u, \tilde{d}_d, \tilde{d}_s)$,

$$d_\rho^{\text{EDM+CEDM+}\theta} = \frac{2\pi^2}{m_\rho^3}(-\langle\overline{q}q\rangle)\left[-\frac{\chi e_- m_* \bar{\theta}}{2} + d_- + \frac{\chi m_0^2}{12}\tilde{d}_-(e_+ - e_-\tilde{m}) \right. \\ \left. + \frac{\chi m_0^2}{2}\tilde{d}_s e_- \frac{m_*}{m_s} + \frac{1}{2}\left(\kappa - \frac{\xi}{2}\right)(e_-\tilde{d}_+ - e_+\tilde{d}_-) \right]. \tag{22}$$

This result parameterizes the effect of all the dimension four and five sources, including the θ-term which was considered previously in [18].

The numerical values for the condensates were obtained in [23], [24] and [14],

$$m_0^2 = 0.8\ \text{GeV}^2\ [23] \tag{23}$$
$$\chi = -5.7 \pm 0.6\ \text{GeV}^{-2}\ [24] \tag{24}$$
$$\kappa = -0.34 \pm 0.1\ [14] \tag{25}$$
$$\xi = -0.74 \pm 0.2\ [14] \tag{26}$$

Note that with these values the combination $(\kappa - \xi/2)$ actually vanishes within the specified precision. Substituting the numerical values for the condensates, masses, and charges, we obtain a final result for the EDM of the ρ-meson:

$$d_\rho = \frac{4.4 \cdot 10^{-3}}{1\text{GeV}}\bar{\theta} + 0.51(d_u - d_d) - 0.13e(\tilde{d}_u - \tilde{d}_d) - \frac{3.5\text{MeV}}{m_s}e\tilde{d}_s \tag{27}$$

Here we have used $m_u = 4.5$ MeV, and $m_d = 9.5$ MeV. We will comment on error estimates in Section IV.

It is interesting to observe that the central point for the QCD sum rule prediction for d_ρ induced purely by the EDMs of the quarks is essentially half the prediction of the naive non-relativistic quark model,

$$d_\rho^{NQM} = d_u - d_d \tag{28}$$

Nonetheless, given the relatively large error bounds on (27), and noting that smaller values of m_u and m_d will bring the two results closer together, we feel that the results are in reasonable agreement.

The contributions of up- and down-quark CEDMs are not, in fact, any smaller than those of the quark EDMs. This may be seen in a simple example of supersymmetric CP-violation due to a squark-gluino loop. The induced EDM operators will then be proportional to the charges of the quarks, thus bringing in an additional factor of $e/3$ to the contribution induced by d_d, leading to a result of the same order as the CEDM contributions. Our result for d_ρ(CEDMs) is larger than one would expect from power-counting rules such as "naive dimensional analysis" [25] which are often applied to estimate d_n(CEDMs). The importance of CEDM contributions in the QCD sum rule approach follows directly from the values of the χ and especially m_0^2 condensates. It seems clear that the estimate within naive dimensional analysis corresponds to a group of perturbative diagrams in which the gluon from $\tilde{d}_q$ is attached to a quark line. This would correspond to an $O(\alpha_s/(4\pi))$ suppression relative to the leading order OPE terms considered here.

Perhaps the most interesting point to make here is the significance of the contribution arising from the strange quark CEDM operator. The $1/m_s$ suppression is in fact fictitious as we expect $d_q \sim m_q$. When this ansatz is assumed, the contribution of the s-quark CEDM becomes even larger than that of the down quark!

However, the presence of the theta term is expected to be numerically dominant, as it corresponds to a dimension four operator and does not experience any suppression by a heavy mass scale. This poses a serious problem for any model of CP-violation other than Kobayashi-Maskawa, as corrections to the theta term are normally large and need to be cancelled by extreme fine-tuning of the "initial condition" for theta. Currently, the accepted recipe to avoid a "θ-dominated" EDM is to assume Peccei-Quinn symmetry. This mechanism is apparently a necessity for any SUSY model operating with CP-SUSY phases around the electroweak scale.

The existence of this mechanism brings an additional contribution to the EDM, not contained in eq. (22). PQ symmetry, although allowing the axion to set $\theta_{\text{QCD}} = 0$, still leads to CP violating terms due to linear contributions to the axion potential [26]. In particular, the axion potential has the form $V \sim -\theta^2 K - 2\theta K'$, where

$$K = i\left\langle \frac{\alpha_s}{8\pi} G\tilde{G}, \frac{\alpha_s}{8\pi} G\tilde{G} \right\rangle \tag{29}$$

is the topological susceptibility, and

$$K' = i\left\langle \frac{\alpha_s}{8\pi} G\tilde{G}, -\frac{i}{2} d_q \overline{q} G\sigma\gamma_5 q \right\rangle \tag{30}$$

are correlators arising from the CEDM sources. This linear shift in the axion potential then leads to an "induced" θ-term with coefficient

$$\theta_{\rm ind} = -\frac{K'}{K} = \frac{m_0^2}{2} \sum_{q=u,d,s} \frac{d_q}{m_q}. \tag{31}$$

This result follows by evaluating the correlators in the manner described in [27] (see also [28]). The value of $\theta_{\rm ind}$ is, of course, independent of the axion coupling constant, and of the particular manner in which PQ symmetry is implemented. Consequently, we find an additional vacuum contribution to the EDM in (11) and (12) of the form,

$$\langle \overline{u}\gamma_5 u\rangle_\theta + \langle \overline{d}\gamma_5 d\rangle_\theta = i\theta_{ind}\langle \overline{q}q\rangle = \frac{i}{2} m_0^2 \left(\sum_{q=u,d,s} \frac{d_q}{m_q} \right) \langle \overline{q}q\rangle. \tag{32}$$

Including this expression[2] in (22) we observe the complete cancelation of the term proportional to the strange quark CEDM, while this new vacuum source of $\tilde{d}_u$ and $\tilde{d}_d$ is combinatorially less suppressed than the direct contributions appearing in (22). The final result then has the form

$$\begin{aligned} d_\rho^{\rm EDM+CEDM+PQ} = & \frac{2\pi^2}{m_\rho^3}(-\langle \overline{q}q\rangle) \left[d_- + \frac{\chi m_0^2}{12} \tilde{d}_-(e_+ - e_-\tilde{m}) \right. \\ & \left. + \frac{\chi m_0^2}{4} (\tilde{d}_+ - \tilde{d}_-\tilde{m}) e_- + \frac{1}{2}\left(\kappa - \frac{\xi}{2}\right)(e_-\tilde{d}_+ - e_+\tilde{d}_-) \right]. \end{aligned} \tag{33}$$

Inserting numerical values for the condensates, masses, and charges as before, we find:

$$d_\rho = 0.51(d_u - d_d) - 0.34e(\tilde{d}_u - \tilde{d}_d) - 0.58e(\tilde{d}_u + \tilde{d}_d). \tag{34}$$

We see that in the presence of PQ symmetry, the contribution from the CEDM sources is even more pronounced.

IV. DISCUSSION

We have studied the EDM of the ρ-meson induced by CP-odd operators of dimension four and five. We find that, at leading order, QCD sum rules predict that d_ρ induced by the

[2]Note that this contribution only affects terms proportional to χm_0^2 as the contributions proportional to ξ and κ are subleading in the EDM induced by θ [18].

quark EDMs is within a factor of two of the prediction of the nonrelativistic quark model. Moreover, a conservative estimate of the error, accounting for higher dimension operators, neglected intermediate states, and single pole contributions, would suggest that this factor of two could easily be accomodated within the precision of our estimates. This precision could of course be improved by including next-to-leading order corrections in the OPE, and also accounting for the single pole contributions on the phenomenological side. An important advantage of the QCD sum rules approach to the calculation of hadronic EDMs over other methods is the proportionality of the result to $d_q\langle\overline{q}q\rangle$ which is normalization-point independent as in most of the models we have $d_q \sim m_q$. The uncertainty related with the poor knowledge of m_q for the light quarks is thus significantly reduced.

We find that d_ρ induced by the colour EDM operators is actually comparable in magnitude with d_ρ(EDMs). There is no specific mechanism of suppression which can be attributed to d_ρ(CEDMs). This conclusion supports an estimate of d_n made in Ref. [16]. We also wish to stress the numerical importance of the color EDM operator of the strange quark, whose contribution is of the same order as that arising from the up and down CEDM operators.

Peccei-Quinn symmetry, which removes $\theta \sim 1$, is an apparent necessity for any model with large CP-violating phases other than Kobayashi-Maskawa. When the PQ mechanism is active, the axion vacuum experiences a linear shift induced by CEDMs. We find that the contribution of the up and down quark CEDMs in this case becomes even larger, whereas the contribution of the strange quark CEDM is completely canceled at this order. This is an important result which may well prove generic and apply also to the experimentally relevant case of the neutron EDM. This important problem will be addressed in a subsequent publication [19].

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